Rossmoyne Senior High School

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | | |

In words _____

_____

Circle your Teacher's Name:

Mrs Alvaro	Mrs Bestall	Mrs Fraser-Jones
Mr Gibbon	Mrs Greenaway	Mr Koulianos
Mr Luzuk	Mrs Murray	Mr Tanday

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ____

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	55	35
Section Two: Calculator-assumed	12	12	100	95	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	8	
9	7	
10	4	
11	9	
12	10	
13	6	
14	10	
15	8	
16	7	
17	10	
18	7	
19	10	
S2 Total	95	
S2 Wt (×0.6633)	65%	

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Section Two: Calculator-assumed **65% (95 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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Question 8 **(8 marks)**

A small body moving in a straight line has an initial velocity of 15 cm/s as it leaves point P. The acceleration of the body at time t seconds is $6 - 1.5t$ cm^2/s, $t \geq 0$.

(a) Determine the displacement of the body relative to P after 2 seconds. (4 marks)

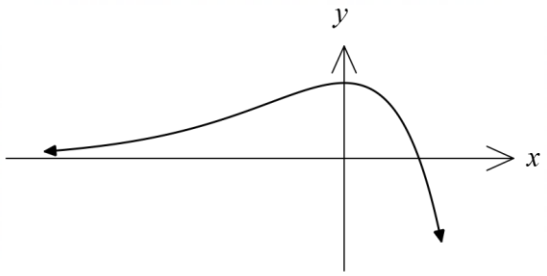
(b) Determine the maximum velocity of the body. (2 marks)

(c) Determine the maximum displacement of the body relative to P. (2 marks)

Question 9 **(7 marks)**

Let $f(x) = (5 - x)e^{0.2x}$.

The graph of $y = f(x)$ is shown at right.



(a) Use calculus to determine the coordinates of the stationary point and justify that the stationary point is a local maximum. (5 marks)

(b) Use calculus to determine the coordinates of the point of inflection. (2 marks)

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Question 10 **(4 marks)**

Rahul has been offered a sales position at a car dealership. His weekly pay will consist of a retainer of $260 and a commission of $600 for each new car sold. The following table shows the probability of him selling specific numbers of cars every week.

n	0	1	2	3	4	5	6
$P(N = n)$	0.1	0.32	0.2	0.15	0.1	0.08	0.05

(a) Explain why the table above is considered a PDF. (2 marks)

(b) Calculate Rahul's expected weekly pay. (2 marks)

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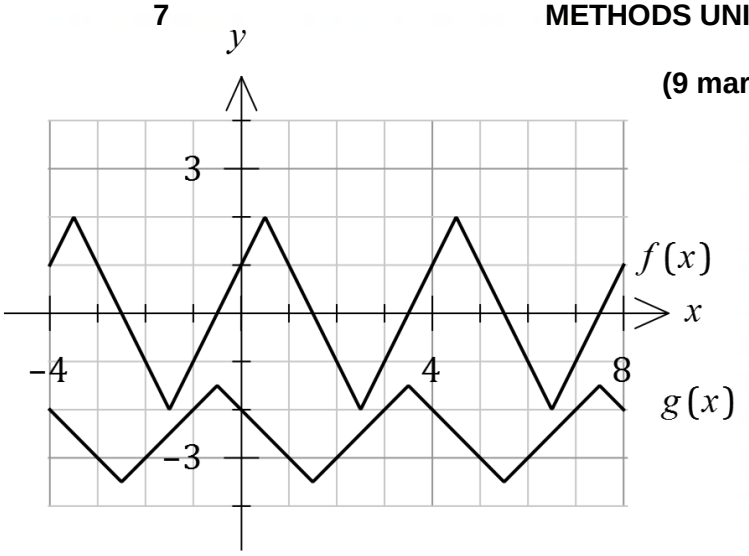
Question 11 **(9 marks)**

The graphs of the continuous functions $y = f(x)$ and $y = g(x)$ are shown at right.



(a) Evaluate the derivative of $g(x)f(x)$ at $x = 2$. (3 marks)

(b) Evaluate the derivative of $g\big(f(x)\big)$ at $x = -3$. (3 marks)

(c) Evaluate the derivative of $\dfrac{f'(x)}{g(x)}$ at $x = 5$. (3 marks)

Question 12 **(10 marks)**

A full water tank takes 38 seconds to empty. The volume V litres of water in the tank, t seconds after emptying began, is changing at a rate given by

$$\frac{dV}{dt} = \sqrt[3]{9t+1} - 7, \qquad 0 \leq t \leq 38.$$

(a) Determine the initial rate of change of volume. (1 mark)

(b) Use the increments formula to estimate the volume of water that empties from the tank during the first one-third of a second. (3 marks)

(c) Determine the initial volume of water in the tank. (3 marks)

See next page

(d) Determine the time, to the nearest 0.01 second, when the tank is half full. (3 marks)

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Question 13 **(6 marks)**

A bag contains three blue and six green balls. Two balls are drawn at random and in succession from the bag. At each draw, if the ball is blue it is replaced in the bag, and otherwise the ball is not replaced. Let X be the number of blue balls drawn.
Construct a probability distribution table for X, using exact values.

See next page

Question 14 **(10 marks)**

The following table shows the probability distribution of a discrete random variable X, where k is a constant.

x	-2	0	1	3
$P(X = x)$	$4k^2$	0.15	$2k$	0.1

(a) Determine the value of k. (3 marks)

(b) Determine $E(X)$. (2 marks)

(c) Given that $\text{Var}(X) = 2.31$, determine the following for the discrete random variable Z:

(i) $E(Z)$ when $Z = 5X - 3$. (1 mark)

(ii) $\text{Var}(Z)$ when $Z = \frac{X}{3} + 2$. (2 mark)

(iii) The standard deviation of Z when $Z = 5(2 - X)$. (2 mark)

Question 15 **(8 marks)**

The concentration of a drug in the plasma of a monkey, C micrograms per litre, t hours after being administered, can be modelled by $C = C_0 e^{kt}$, where C_0 and k are constants.
Each dose of the drug immediately increases the existing concentration by $430\ \mu g/L$ ($C_0 = 430$), and the concentration of the drug is known to halve every 2 hours and 40 minutes.

A monkey, with no existing trace of the drug, was administered a first dose at $8{:}00$ am.

(a) Use the model to determine the rate of change of concentration of the drug in the monkey's plasma later that morning at $10{:}40$ am. (4 marks)

An additional dose is administered every time the concentration falls to $130\ \mu g/L$.

(b) Determine the expected time of day, to the nearest minute, that the third dose will be administered to the monkey. (4 marks)

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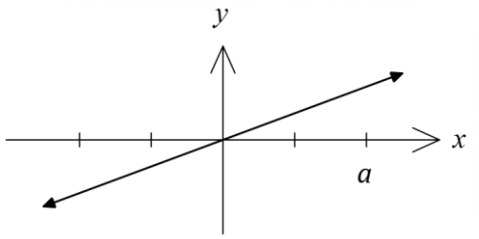
Question 16 **(7 marks)**

(a) Consider the function $f(x) = mx$, where m is a constant.
The graph of $y = f(x)$ is shown at right, a is a constant and

$$\int_0^a f(x)\,dx = 5.$$



Determine the value of

(i) $\int_{-a}^{a} f(x)\,dx.$ (1 mark)

(ii) $\int_0^a 2f(x-a)\,dx.$ (2 marks)

(b) The polynomial function $g(x)$ is such that $\int_{-2}^{3} g(x)\,dx = 8.$

Determine the value of $\int_{-2}^{1} (2x + g(x))\,dx + \int_1^3 (g(x) - 1)\,dx.$ (4 marks)

Question 17 **(10 marks)**

A bus is scheduled to arrive at a particular bus stop at 8:41 am.
It is equally likely to arrive during any minute between 8:39am and 8:46am.

T is defined as the number of whole minutes that the bus arrives earlier or later than the scheduled time.

(a) Explain why T is defined as a discrete random variable **and** why the domain of T is {–2, –1, 0, 1, 2, 3, 4, 5}. (2 marks)

(b) Write down the probability distribution for *T*. (2 marks)

(c) Determine the probability that a bus:

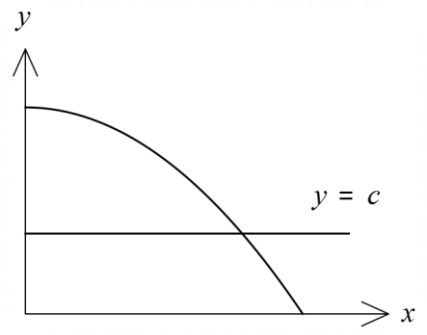
(i) arrives on time. (1 mark)

(ii) arrives at 8:42 am, if it is late. (2 marks)

(iii) arrives less than 3 minutes late, if it is not on time. (3 marks)

Question 18 **(7 marks)**

The line $y = c$ divides the area in the first quadrant under the curve $y = 16 - x^2$ into two equal halves, as shown in the diagram.



Determine, with reasoning, the value of c.

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Question 19 **(10 marks)**

A small body moves in a straight line with velocity v cm/s at time t s given by

$$v(t) = 11 + 4\sin\left(\frac{\pi t}{10}\right) - 6\sin\left(\frac{\pi t}{5}\right), \qquad t \geq 0.$$

(a) By viewing the graph of the velocity function on your calculator, or otherwise, state the minimum velocity of the body for $t \geq 0$ to the nearest 0.01 cm/s, and hence explain why the distance travelled by the body in any interval of time will always be the same as the change in displacement of the body. (2 marks)

(b) Determine the distance travelled by the body between $t = 0$ and $t = 20$. (2 marks)

The distance travelled (x cm) by the body in any 10 second interval from $t = T$ to $t = T + 10$ is given by the function $x(T) = a + b\cos\left(\frac{\pi T}{10}\right)$.

(c) Determine the value of the constant a and the value of the constant b. (2 marks)

End of questions

(d) During the first 35 seconds, there is a 10 second interval in which the distance travelled by the body is a minimum. Using calculus methods, determine when this interval occurs and justify that the distance is a minimum. (4 marks)

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Supplementary page

Question number: _____